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THE CALCULATION OF STREAMLINE DATA FOR BOUNDARY LAYER INPUT,(U)
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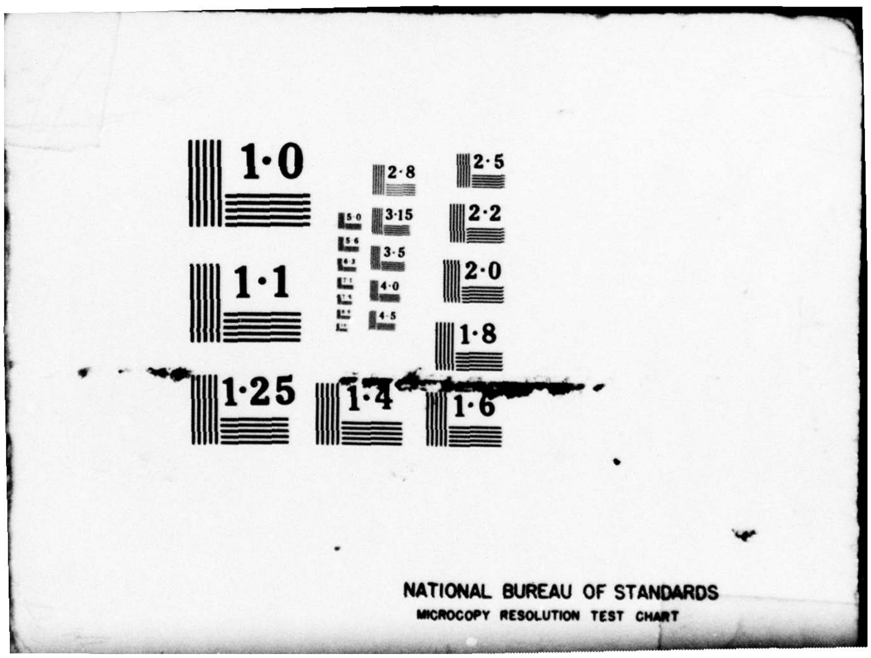
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THE CALCULATION OF STREAMLINE DATA FOR BOUNDARY LAYER INPUT

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DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Md. 20084

LEVEL II

THE CALCULATION OF STREAMLINE DATA FOR BOUNDARY LAYER INPUT

Charles W. Dawson

and

Janet S. Dean

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COMPUTATION, MATHEMATICS, AND LOGISTICS DEPARTMENT
DEPARTMENTAL REPORT

January 1974 CMD-4-74

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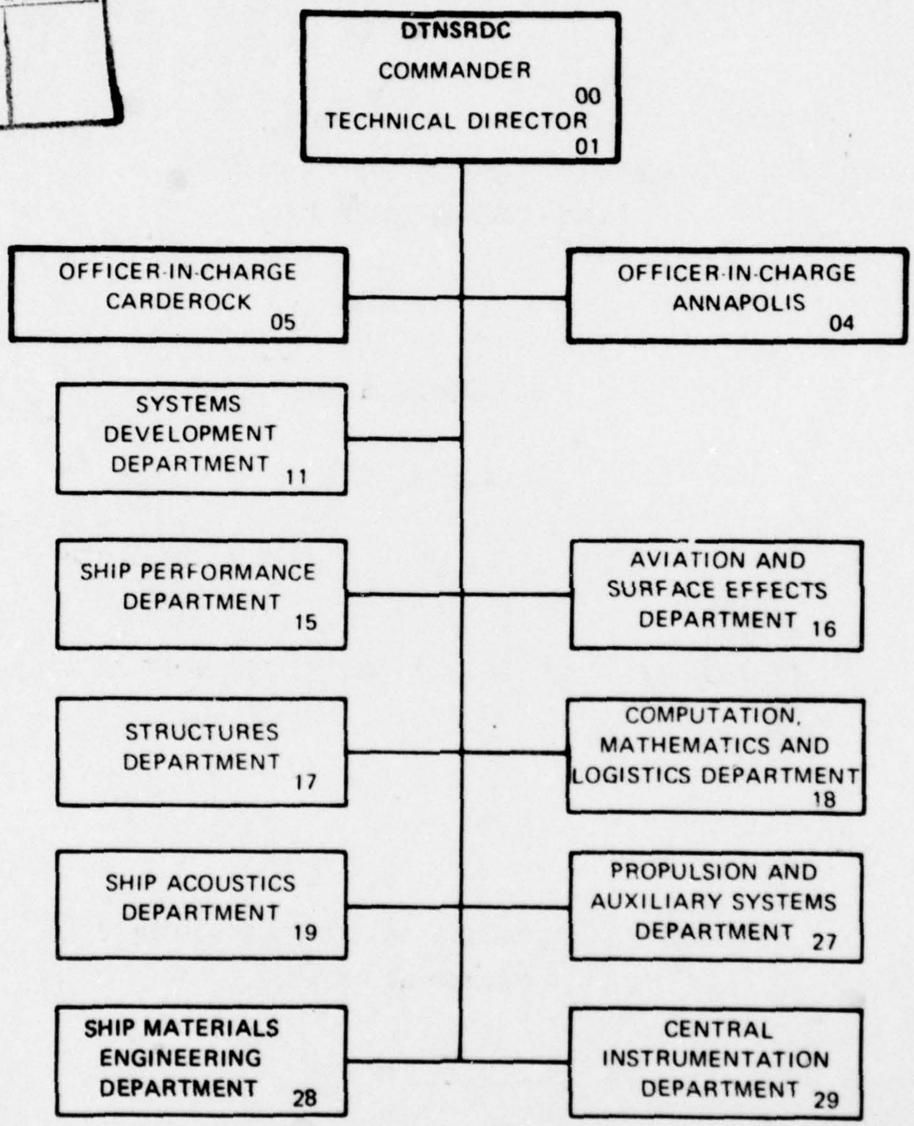

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THE CALCULATION OF STREAMLINE DATA FOR BOUNDARY LAYER INPUT

Charles W. Dawson
and
Janet S. Dean

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TABLE OF CONTENTS

LIST OF FIGURES

ABSTRACT

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ADMINISTRATIVE INFORMATION

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I. INTRODUCTION

The value of boundary layer calculations for problems of two-dimensional or axisymmetric flow has been amply demonstrated. It is generally believed that three-dimensional boundary layer calculations will be of similar value. A number of methods have recently been developed for calculating three-dimensional boundary layers with small cross flow. These methods require that the values of the velocity and of the geodesic curvatures of both the equipotential lines and the streamlines be specified along streamlines on the body surface.

The XYZ Potential Flow Program has recently been modified to provide input data for three-dimensional boundary layer calculations. These modifications are part of a larger effort to improve the XYZ Potential Flow Program by including curved surface elements and linear source terms. A hybrid program including parts of the older and newer versions is being made available now because of the great interest shown in three-dimensional boundary layer calculations. This report is designed to be used with NSRDC Report 3892[1] on the XYZ Potential Flow Program and is not self-contained.

Perhaps it would be wise to emphasize here that small cross flow boundary layer calculations are still of an experimental nature. For example, the starting conditions and the point of transition from laminar to turbulent flow must be correctly specified if accurate results are to be obtained. These conditions are generally not known and few people have much experience at guessing them. Thus, this report is directed more to those interested in developing boundary layer theory than to those doing design studies.

---

[1] Dawson, Charles W. and Dean, Janet S., "The XYZ Potential Flow Program," NSRDC Report 3892, June 1972.

II. THREE-DIMENSIONAL BOUNDARY LAYER METHODS

Figure 1 shows a three-dimensional body with two streamlines. Note that the streamlines converge or diverge and curve as they pass along the body side. The convergence or divergence of the streamlines can be modeled for each streamline by an axisymmetric body as shown in Figure 2. In other words, when the radius of the axisymmetric body changes in the right way, the streamlines will converge or diverge at the same rate as they do at the particular streamline on the three-dimensional body. The rate of convergence will change from streamline to streamline, so a different body is needed for each streamline. Thus as a first approximation the boundary layer on the three-dimensional body may be obtained by solving a set of axisymmetric boundary layer problems.

The curvature of the streamlines on the body surface indicates the presence of a pressure gradient across the streamlines so that there will be a cross flow within the boundary layer. When the usual small cross flow approximation is used, there is an equation for the streamwise boundary layers and an equation for the cross flow. The cross flow depends upon the streamwise flow but the streamwise flow is usually treated as being independent of the cross flow. Thus, the streamwise flow computed from the small cross flow approximation will be the same as that computed by an axisymmetric program. A computer program of this type developed at Douglas Aircraft Company by Cebeci, Mosinskis and Kaups[2] is available at NSRDC.

---

[2] Cebeci, T; Mosinskis, G.J.; Kaups, K, "A General Method for Calculating Three-Dimensional Incompressible Laminar and Turbulent Boundary Layers I. Swept Infinite Cylinders and Small Cross Flow," Douglas Aircraft Company Report No. MDC J5694, Nov 1972.

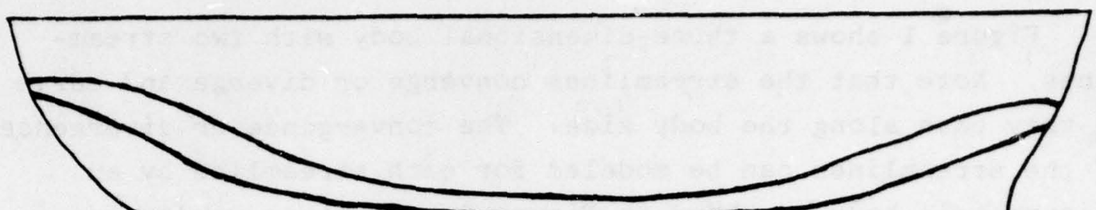

Figure 1 - Streamlines on a Three-dimensional Body

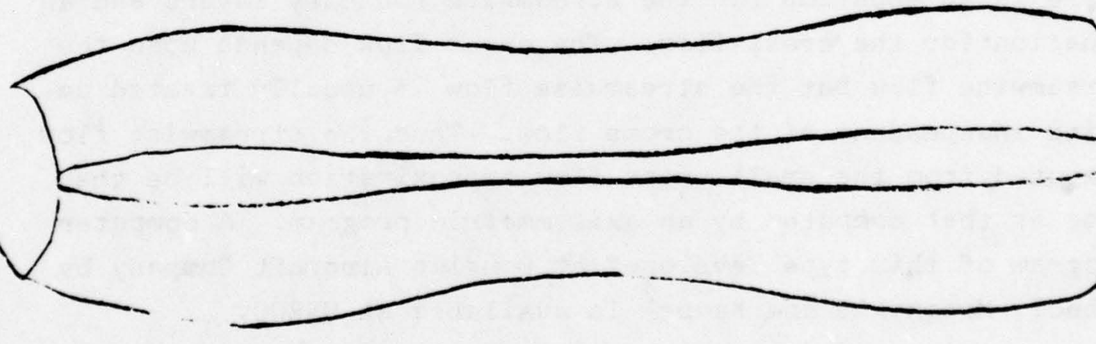

Figure 2 - Axisymmetric Pipe Model of a Three-dimensional Body for One Streamline

III. NATURE OF THE PROBLEM

The XYZ Potential Flow Program computes an approximate solution to the problem of three-dimensional potential flow over an arbitrarily shaped body. The body surface is represented by a set of plane quadrilaterals. In the solution the values of the velocity are computed at the centroids of the quadrilaterals. A subprogram which computes the on-body streamlines is given only the quadrilaterals and the velocity at the centroids. The earlier subprogram assumed that the velocity was constant in each of the quadrilaterals. The resulting streamline data was quite rough and did not include any information on the curvature of the streamlines or equipotential lines. The new subprogram uses data from two of the neighboring quadrilaterals and thus can include linear terms in the approximation to the velocity. As a result the streamline data is much smoother and the geodesic curvatures of the streamlines and equipotential lines can be computed. Using the geodesic curvatures the metric, or effective radius of the body, along the streamline is computed. This data can then be used with either a small cross flow boundary layer program or, ignoring cross flow, with an axisymmetric boundary layer program.

IV. RESTRICTIONS ON THE INPUT

The input is the same as that described in NSRDC Report 3892. However, there are four restrictions. These restrictions are required because the quadrilaterals must be organized so that neighboring quadrilaterals are readily available. The quadrilaterals are organized in groups of four about a common corner point as in Figure 3. Thus there must be an even number of quadrilaterals in both the N and M directions in each of the sections of the body.

The sides between quadrilaterals are used to define the quadrilateral coordinate systems and serve as the axis of rotation when the surface is flattened to facilitate the numerical differentiation of the velocity. Thus, the common corner point of a group of four quadrilaterals must not coincide with any other corner point. The third restriction is that each set of four quadrilaterals must have at least seven distinct corner points to allow a curved surface to be fitted to the points.

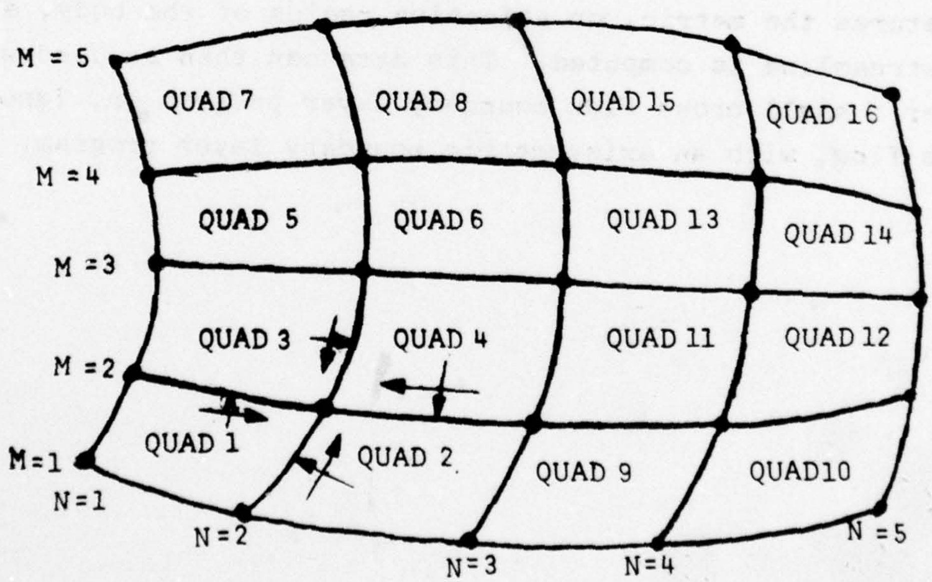

Figure 3 - Ordering of the Quadrilaterals
For A Right Handed Coordinate System

Thus only two of the four quadrilaterals may degenerate into triangles by having two of their corner points coincide.

The last restriction also is a result of the way the curved surface is fitted to the points. The angle between the normal vectors of two quadrilaterals in a group of four must be less than 90° and perferably less than 45°. If a sharp edge is required the input should be arranged so that the edge is along the boundary of the groups of four and not through the center.

V. COMPUTATION OF THE PARTIAL DERIVATIVES OF THE VELOCITY

The quadrilateral is placed in a standard position with an upper neighbor and a right neighbor as shown in Figure 2. The other neighbors are ignored. The quadrilateral coordinate system is designated by x_q, y_q, z_q, the right neighbor coordinate system is designated by x_r, y_r, z_r and the upper neighbor coordinate system by x_u, y_u, z_u. Note that x_q is parallel to the upper boundary of the quadrilateral and x_r is parallel to the right boundary of the quadrilateral.

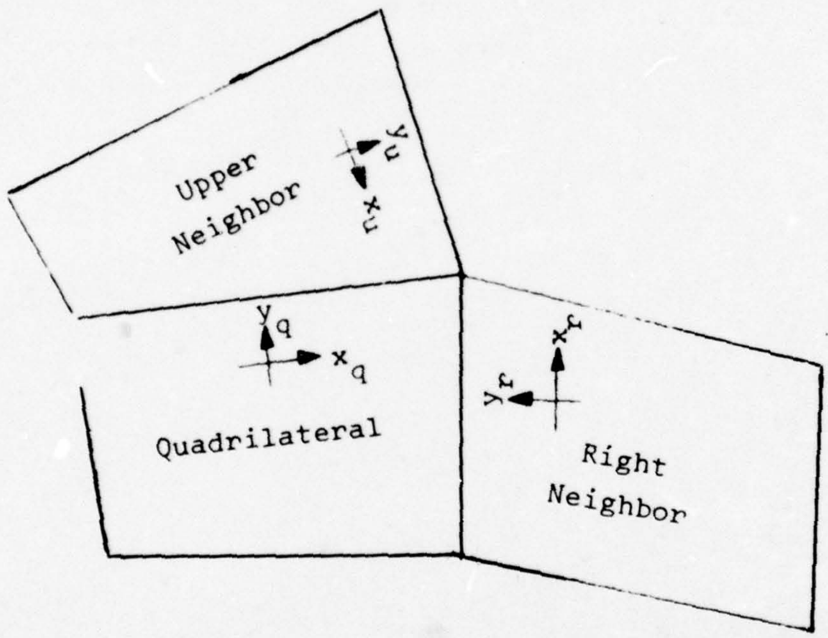

Figure 4 - Standard Position for a Quadrilateral and its Neighbors

In general, the neighbors will not be in the same plane as the quadrilateral. Thus, the first task is to compute appropriate velocity and position vectors for the neighbors in the quadrilateral coordinate system as though they were in the quadrilateral plane. This is done as follows:

1. The cosine of the angle of rotation between the quadrilateral plane and the plane of the upper neighbor ($\cos\theta_u$) is found from the dot product of their unit normal vectors.

2. The upper neighbor velocity vector is transformed into the quadrilateral coordinate system. The y_q component is divided by $\cos\theta_u$ to correct for the effect of surface curvature and the normal component is set to zero. The result is as though the velocity vector had been rotated about the boundary between the quadrilateral and the upper neighbor into the plane of the quadrilateral.

3. The coordinates of the centroid of the upper neighbor are transformed into the quadrilateral coordinate system. An approximate correction for the effect of surface curvature is made to the y_q coordinate.

$$y_q \text{ corrected} = \frac{1}{6}\left[4\,(y_q^2 + z_q^2)^{\frac{1}{2}} + y_q + y_q/\cos\theta_u\right] \tag{1}$$

This equation was obtained as follows. The arc length along a curve in the y, z plane is

$$\ell = \int_{y_0}^{y_1} \left(1 + \left(\frac{dz}{dy}\right)^2\right)^{\frac{1}{2}} dy$$

When approximated by Simpson's Rule, ℓ becomes

$$\ell = \frac{[y_1 - y_0]}{6}\left\{\left[1 + \left(\frac{dz}{dy}\right)^2_{y_0}\right]^{\frac{1}{2}} + 4\left[1 + \left(\frac{dz}{dy}\right)^2_{y_{\frac{1}{2}}}\right]^{\frac{1}{2}} + \left[1 + \left(\frac{dz}{dy}\right)^2_{y_1}\right]^{\frac{1}{2}}\right\}$$

Now $y_1 - y_0 = y_q$

$$\left(\frac{dz}{dy}\right)_{y_0} = 0$$

$$\left(\frac{dz}{dy}\right)_{y_1} = \tan\theta_u \quad \text{thus} \quad \left[1 + \left(\frac{dz}{dy}\right)^2_{y_1}\right] = 1/\cos\theta_u$$

Since $(\frac{dz}{dy})_{y_{\frac{1}{2}}}$ is not known, the term

$[y_1 - y_0]\left[1 + (\frac{dz}{dy})^2_{y_{\frac{1}{2}}}\right]^{\frac{1}{2}}$ is replaced by the chord length

$(y_q^2 + z_q^2)^{\frac{1}{2}}$. Thus equation (1) is obtained, and the normal component can now be ignored.

4. The cosine of the rotation angle $(\cos\theta_r)$ between the planes of the quadrilateral and its right neighbor is computed from the dot product of their unit normal vectors.

5. The right neighbor velocity vector is transformed into the right neighbor coordinate system and the y_r component is divided by $\cos\theta_r$. The resulting vector is transformed into the quadrilateral coordinate system and the normal component is set to zero. As with the upper neighbor this has the effect of rotating the vector about the right boundary into the quadrilateral plane.

6. The vector from the quadrilateral centroid to the right neighbor centroid is transformed into the right neighbor coordinate system. The y_r component is corrected for surface curvature and for the transformation to the quadrilateral coordinate system.

$$y_r \text{ corrected} = [4(y_r^2 + z_r^2)^{\frac{1}{2}} + y_r + y_r/\cos\theta_r]/6\cos\theta_r \quad (2)$$

The normal component is set to zero and the resulting vector is transformed into the quadrilateral coordinate system.

The velocity and coordinates are now known for three points in a plane. Thus the two tangential components of the velocity may be approximated by

$$U = U_q + U_1 x_q + U_2 y_q$$
$$V = V_q + V_1 x_q + V_2 y_q \quad (3)$$

where U is the x_q component of the velocity

V is the y_q component of the velocity

The equations from the three points readily yield the three unknowns in each approximation. Thus (U_1, V_1) and (U_2, V_2) which are the approximate partial derivatives of the velocity with respect to x_q and y_q are obtained.

VI. COMPUTATION OF POINTS ON A STREAMLINE

The streamline computation begins at a given point in a specified quadrilateral with the velocity vector $\vec{V} = (U,V)$ given by equations (3). A stream function is desired which will have a constant value of zero along the streamline. Since the divergence of $\vec{V}$ is not zero, an additional function ρ must be found such that the divergence of $\rho\vec{V}$ is zero. Then for a particle following a streamline:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{V}{U} = \frac{\rho V}{\rho U}$$

Thus a new vector field is constructed whose streamlines are identical to those of the velocity field but whose divergence is zero. The function ρ is given by the series:

$$\rho(x_q,y_q) = 1 - \frac{U_q\,(U_1+V_2)}{U_q^2+V_q^2}\,x_q - \frac{V_q(U_1+V_2)}{U_q^2+V_q^2}\,y_q + \ldots \tag{4}$$

The resulting stream function is:

$$SF(x_q,y_q) = SF_0 - V_q\,x_q + U_q\,y_q - \left[V_1 - \frac{U_qV_q(U_1+V_2)}{U_q^2+V_q^2}\right]\frac{x_q^2}{2} \tag{5}$$

$$+ \left[U_2 - \frac{U_qV_q(U_1+V_2)}{U_q^2+V_q^2}\right]\frac{y_q^2}{2} + \left[U_1 - \frac{U_q^2(U_1+V_2)}{U_q^2+V_q^2}\right]x_q\,y_q$$

$$+ \ldots$$

The constant value SF_0 is chosen so that the stream function is zero at the specified point. Since the velocity is known only through the linear terms it is consistent to truncate the stream function after the quadratic terms.

Values of SF at the four corner points of the quadrilateral are now computed. By comparing the signs of SF at adjacent corners, the sides through which the streamline passes are determined. If the streamline passes through a side, the value of SF is computed at the midpoint of the side. From these three points the intersection point is computed from a three point interpolation formula.

The parameter t $(0 \leq t \leq 1)$ is defined so that:

$$\begin{aligned} x &= x_1 + t\,(x_3 - x_1) \\ y &= y_1 + t\,(y_3 - y_1) \end{aligned} \tag{6}$$

Here x, y is the intersection point.

x_1, y_1 is one corner point.

x_3, y_3 is the other corner point.

x_2, y_2 is the middle point.

Then:

$$0 = SF(x,y) = 2\,SF_1\,(t-\tfrac{1}{2})(t-1) - 4\,SF_2\,t(t-1) + 2\,SF_3\,t(t-\tfrac{1}{2}) \tag{7}$$

The root of this equation is chosen so that $0 \leq t \leq 1$. Note that if there had been two roots between 0 and 1, then SF would have the same sign at both corner points and those intersection points would be ignored completely.

In general, two intersections will be found for the entire quadrilateral, one where the streamline enters and one where it leaves. It is possible, however, for there to be four intersection points. The next point on the streamline is chosen from the intersection points as follows. A quantity Q is computed for each intersection point by taking the dot product of the

vector from the starting point to the intersection point with the velocity vector at the centroid of the quadrilateral and then multiplying it by the direction sign. (The direction sign is +1 if the streamline is being traced downstream and -1 if it is being traced upstream.) The intersection point with the maximum positive value of Q is chosen. However, if the largest Q is less than or equal to zero, none of the points is acceptable and the program searches for another quadrilateral through which the streamline might pass.

Since each of the points representing a streamline is located at the boundary between two quadrilaterals, the values of the velocity and geodesic curvatures at each point can be computed separately for the two quadrilaterals. The average of the two values at a point is taken as the value at that point.

The streamline is first traced in the downstream direction and then in the upstream direction. After the entire streamline has been obtained the arc length along the streamline is computed so that it starts at zero at the upstream end. Points which are very close together are combined. Such points occur when the streamline just cuts across a corner of a quadrilateral. If the distance between two points is less than 1/8 the distance between their neighbor in front and their neighbor in back, they are combined and average values of their velocity, position and curvatures are used for the new point.

VII. COMPUTATION OF THE GEODESIC CURVATURES (K_1 AND K_2) AND THE METRIC COEFFICIENT H_2

The unit vector tangent to the streamline is

$$\vec{T} = \frac{\vec{i}\,U + \vec{j}\,V + \vec{k}\,W}{(U^2 + V^2 + W^2)^{\frac{1}{2}}}$$

where U, V, W are the components of the velocity vector.

$\vec{i}$, $\vec{j}$, $\vec{k}$ are unit vectors in the three coordinate directions.

The curvature of the streamline is defined by

$$\vec{K} = d\,\vec{T}/d\ell$$

where ℓ is the arc length along the streamline. The geodesic curvature is the component of $\vec{K}$ on the body surface

$$K_g = \vec{K} \cdot (\vec{T} \times \vec{N})$$

where $\vec{N}$ is the unit normal vector to the surface.

For a plane quadrilateral and with vectors written in the quadrilateral coordinate system we have

$$\vec{T} = (\vec{i}\,U + \vec{j}\,V) \,/\, (U^2+V^2)^{\frac{1}{2}}$$

$$\frac{d}{d\ell} = (U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y}) \,/\, (U^2+V^2)^{\frac{1}{2}}$$

$$\vec{K} = (\vec{i}\,V - \vec{j}\,U)\,[U(V\,U_x - U\,V_x) + V(V\,U_y - U\,V_y)] \,/\, (U^2+V^2)^2$$

where subscripts x and y denote partial differentiation

$$\vec{T} \times \vec{N} = (-\vec{i}\,V + \vec{j}\,U) \,/\, (U^2+V^2)^{\frac{1}{2}}$$

Thus K_2, the geodesic curvature for the streamline is obtained.

$$K_2 = [U(U\,V_x - V\,U_x) + V(U\,V_y - V\,U_y)] \,/\, (U^2+V^2)^{\frac{3}{2}} \qquad (8)$$

The geodesic curvature for the equipotential lines, K_1, is found by simply replacing U by V and V by -U in equation 8.

$$K_1 = [-V(VU_x - UV_x) + U(VU_y - UV_y)] / (U^2+V^2)^{\frac{3}{2}} \qquad (9)$$

The metric coefficient H_2 is computed from the values of K_1. By definition

$$K_1 = \frac{-1}{H_1 H_2} \frac{\partial H_2}{\partial \ell}$$

H_1 is defined to be one along the streamline so:

$$H_2 K_1 = - \frac{\partial H_2}{\partial \ell}$$

This equation is approximated by:

$$H_2(\ell + \Delta\ell) - H_2(\ell) = - \frac{\Delta\ell}{2} [H_2(\ell) K_1(\ell) + H_2(\ell + \Delta\ell) K_1(\ell + \Delta\ell)]$$

or

$$H_2(\ell + \Delta\ell) = H_2(\ell) \left[\frac{2 - \Delta\ell K_1(\ell)}{2 + \Delta\ell K_1(\ell + \Delta\ell)} \right] \qquad (10)$$

where $\Delta\ell$ is the chord length of the segment of the streamline passing through the quadrilateral.

Note that H_2 is determined only to within a multiplication constant. This constant was arbitrarily chosen so that H_2 is one at the specified starting point for the streamline. Thus, for an axisymmetric body H_2 will be proportional to the radius but will not in general equal the radius.

VIII. POSSIBLE DIFFICULTIES

There are several possible difficulties in the computation of a streamline that can cause the program to end the streamline in the wrong place. These are:

1. When the projection of the starting point onto the plane of the starting quadrilateral lies outside the starting quadrilateral, there are two possiblities which may occur. The streamline will be traced in only one direction if it passes through the starting quadrilateral or it will not be traced at all if it does not pass through the starting quadrilateral.

2. If the streamlines are converging from both sides toward the boundary line between two quadrilaterals, it may happen that due to numerical approximations the computed streamline will intersect the boundary. Then the program cannot find another quadrilateral in which to continue the streamline and will end the streamline at that point.

3. The present program does not provide for the continuation of a streamline across a plane of symmetry. Hence, when a streamline intersects a plane of symmetry it will be terminated.

IX. OUTPUT

The edited output includes the following information: (See Appendix B for a sample of edited output.)

1. The problem identification and information from the parameter card.

2. Information concerning each of the quadrilaterals specified by the input. This includes

   a. Warning messages about possible errors. A message "Questionable Point - Poor Fit" replaces the "LARGE D" message described on page 39 of NSRDC Report 3892.

   b. M,N - the indices for the first corner point.

   c. P - the quadrilateral number in the total array of quadrilaterals.

   d. X1, Y1, Z1 - the coordinates given for the first corner point. (Point M,N.)

   e. X2, Y2, Z2 - the coordinates given for the second corner point. (Point M+1,N.)

   f. X3, Y3, Z3 - the coordinates given for the third corner point. (Point M+1,N+1.)

   g. X4, Y4, Z4 - the coordinates given for the fourth corner point. (Point M,N+1.)

   h. XP,YP,ZP - the coordinates computed for the centroid.

   i. XN,YN,ZN - the components of the normal vector.

   j. A - the area of the quadrilateral.

   k. FL - the maximum distance of a corner point from the centroid of the quadrilateral.

1. CZ1,CZ4,CZ5,CZ6 - the coefficients of a local quadratic fit of the body surface. These are not used in the present version of the program except for checking input.

3. Information about the convergence of the iterations for computing the source density. This information includes

a. The sum of the absolute values of the changes in the source density from the last iteration.

b. $\overline{A}$, $\overline{B}_1$, and $\overline{B}_2$ - the extrapolation coefficients computed from the last five iterations. (Once every five iterations.)

c. A message indicating extrapolation has been performed.

4. The edit of the final solution includes, for each of the three basic flows

a. The point number - the same number as the P in Part 2.c of the output.

b. X,Y,Z - the coordinates of the centroid.

c. VX,VY,VZ - the components of the velocity at the centroid.

d. ABS · V - the absolute value of the velocity.

e. CP - the pressure coefficient, $CP = 1 - V^2/V_\infty^2$.

f. The source density.

g. The normal component of the velocity.

5. The edit of the solution for each additional flow includes the same items as for the three basic flows except that items f. and g. are omitted.

6. The output for each on-body streamline includes the following quantities for each of a set of points on the streamline:

a. X,Y,Z - the coordinate of the point.

b. VX,VY,VZ - the components of the velocity.

c. CP - the pressure coefficient.

d. K1,K2 - the geodesic curvatures.

e. H2 - the metric coefficient.

f. L - the arc length along the streamline from the upstream end to the current point (X,Y,Z).

g. V - the absolute value of the velocity.

X. USE OF THE OUTPUT FOR BOUNDARY LAYER CALCULATIONS

Most boundary layer programs require input consisting only of the values of K2, H2, L, and V which are calculated by the XYZ Potential Flow Program and described in Chapter VIII. It might seem then that using this potential flow data to do a boundary layer calculation would be very simple. Unfortunately there are several points which must be considered before proceeding with the boundary layer calculation.

First, the data as it comes from the XYZ Potential Flow Program may be rather rough or bumpy and may not contain enough points for an accurate boundary layer calculation. Thus, it may be necessary to use some curve fitting routine to fit a smoother curve through the data and thereby increase the number of data points.

Another problem is that of specifying the initial velocity profile and the point of transition from laminar to turbulent flow. It should <u>not</u> be assumed a priori that a method which is satisfactory for two-dimensional or axisymmetric problems will be equally satisfactory for small cross flow problems. Experiments with several methods may be necessary to obtain satisfactory results.

Axisymmetric boundary layer programs require the radius of the body as input. Thus, when an axisymmetric boundary layer program is used, the radius of the analogous axisymmetric body must be computed. This radius will be a constant times the H_2 computed by the XYZ program. There are two points to consider in choosing this constant. First, the change in the radius (ΔR) between any two points on the streamline must be less than the arc length ($\Delta \ell$) between them. Secondly, if the initial velocity profile is to be determined by the wedge angle at the leading end of the streamline, then ΔR for the first two points must satisfy the equation

$$\Delta R = \Delta \ell \sin \theta, \tag{11}$$

where θ is the half angle of the wedge. Thus the values of R along the streamline are determined for the entire streamline. Note that the values of R which are determined by equation 11 might not satisfy the first condition that $\Delta R \leq \Delta \ell$ everywhere on the streamline. This may or may not cause trouble,depending upon the way the boundary layer program is coded.

APPENDIX A
DECK SETUP FOR STANDARD RUN OF SAMPLE PROBLEM SPHERE

```
JOBNAME,CM100000,T100,P2.                    * USERS JOB CARD *
CHARGE,CXXX,PPPPPPPPPP,R5,8.                 * USERS CHARGE CARD *
RFL,55000.                                   * CXXX IS USERS ID *
LIMIT,3584.                                  * PPPPPPPPPP IS USERS *
REQUEST,TAPE03,*PF.                          * JOB ORDER NUMBER    *
REQUEST,TAPE04,*PF.
REQUEST,TAPE7,*PF.
ATTACH(PF1,CXXXPF1V4DATE,MR=1)               * ATTACH PROGRAM *
ATTACH(PF2,CXXXPF2V4DATE,MR=1)               * COMPILATIONS   *
ATTACH(PF3,CXXXPF3V1DATE,MR=1)
ATTACH(PF4,CXXXPF4V1DATE,MR=1)
ATTACH(PF7,CXXXPF7V4DATE,MR=1)
ATTACH,MAT,CLIBAMMAT4.                       * OMIT IF MATINS WAS *
LOAD(MAT)                                    * COMPILED WITH PF1  *
PF1.
RFL,45000.
PF2.
RFL,45000.
PF3.
RFL,55000.
PF4.
CATALOG(TAPE03,CXXXT03DATE,ID=PPPPPPPPPP)
CATALOG(TAPE04,CXXXT04DATE,ID=PPPPPPPPPP)
RFL,100000.
PF7.
CATALOG(TAPE7,CXXXT7DATE,ID=PPPPPPPPPP)
EXIT.
           7/8/9     END OF RECORD
         SAMPLE PROBLEM    SPHERE
 12    3 150 150 150    3    .0001
 1.0            .0             .0            1    1    1
  .92388        .0             .38268        1    2    1
 .70711         .0             .70711        1    3    1
  .92388        .38268         .0            2    1    1
  .88808        .32504         .32504        2    2    1
  .67383        .30320         .67383        2    3    1
  .70711        .70711         .0            3    1    1
  .67383        .67383         .30320        3    2    1
  .57735        .57735         .57735        3    3    1
  .0           1.0             .0            1    1    2
  .38268        .92388         .0            1    2    2
  .70711        .70711         .0            1    3    2
  .0            .92388         .38268        2    1    2
  .32504        .88808         .32504        2    2    2
  .67383        .67383         .30320        2    3    2
  .0            .70711         .70711        3    1    2
  .30320        .67383         .67383        3    2    2
  .57735        .57735         .57735        3    3    2
  .0            .0            1.0            1    1    3
  .0            .38268         .92388        1    2    3
```

```
  .0            .70711        .70711       1    3    3
  .38268        .0            .92388       2    1    3
  .32504        .32504        .88808       2    2    3
  .30320        .67383        .67383       2    3    3
  .70711        .0            .70711       3    1    3
  .67383        .30320        .67383       3    2    3
  .57735        .57735        .57735       3    3    3
          7/8/9      END OF RECORD
 1.0            .0            .0                2
  .71           .45           .45               4
  .17           .17           .95               9

          7/8/9      END OF RECORD
        6/7/8/9      END OF FILE
```

APPENDIX B
OUTPUT FOR SAMPLE PROBLEM SPHERE

POTENTIAL FLOW PROGRAM SECTION 1

SAMPLE PROBLEM SPHERE

NO. OF QUADS. = 12
NO. OF SECTIONS= 3
MAX. NO. OF ITERATIONS X FLOW 150 Y FLOW 150 Z FLOW 150
3 PLANES OF SYMMETRY

CONVERGENCE CRITERIA , .00010

SECTION 1

M	X1	X2	X3	X4	XP	XN	A	CZ4
N	Y1	Y2	Y3	Y4	YP	YN	FL	CZ5
P	Z1	Z2	Z3	Z4	ZP	ZN	CZ1	CZ6
1	.10000E+01	.92388E+00	.88808E+00	.92388E+00	.93408E+00	.96748E+00	.12802E+00	-.55519E+00
1	0.	0.	.32504E+00	.38268E+00	.17163E+00	.17887E+00	.27060E+00	-.55928E+00
1	0.	.38268E+00	.32504E+00	0.	.17163E+00	.17887E+00	.38023E-01	-.11684E-01
1	.92388E+00	.88808E+00	.67383E+00	.70711E+00	.80023E+00	.82701E+00	.12715E+00	-.55114E+00
2	.38268E+00	.32504E+00	.67383E+00	.70711E+00	.52041E+00	.53792E+00	.26663E+00	-.55938E+00
2	0.	.32504E+00	.30320E+00	0.	.15846E+00	.16339E+00	.35472E-01	.53791E-02
2	.92388E+00	.70711E+00	.67383E+00	.88808E+00	.80023E+00	.82701E+00	.12715E+00	-.55845E+00
1	0.	0.	.30320E+00	.32504E+00	.15846E+00	.16339E+00	.26663E+00	-.55207E+00
3	.38268E+00	.70711E+00	.67383E+00	.32504E+00	.52041E+00	.53792E+00	.35472E-01	.74945E-02

QUESTIONABLE POINT -POOR FIT .749E-03

2	.88808E+00	.67383E+00	.57735E+00	.67383E+00	.71067E+00	.73490E+00	.12400E+00	-.55338E+00
2	.32504E+00	.30320E+00	.57735E+00	.67383E+00	.46210E+00	.47954E+00	.25207E+00	-.55116E+00
4	.32504E+00	.67383E+00	.57735E+00	.30320E+00	.46210E+00	.47954E+00	.37339E-01	-.11954E-01

SECTION 2

M N P	X1 Y1 Z1	X2 Y2 Z2	X3 Y3 Z3	X4 Y4 Z4	XP YP ZP	XN YN ZN	A FL CZ1	CZ4 CZ5 CZ6
1	0.	.38268E+00	.32504E+00	0.	.17163E+00	.17887E+00	.12802E+00	-.55519E+00
1	.10000E+01	.92388E+00	.88808E+00	.92388E+00	.93408E+00	.96748E+00	.27060E+00	-.55928E+00
5	0.	0.	.32504E+00	.38268E+00	.17163E+00	.17887E+00	.38023E-01	-.11684E-01
1	0.	.32504E+00	.30320E+00	0.	.15846E+00	.16339E+00	.12715E+00	-.55114E+00
2	.92388E+00	.88808E+00	.67383E+00	.70711E+00	.80023E+00	.82701E+00	.26663E+00	-.55938E+00
6	.38268E+00	.32504E+00	.67383E+00	.70711E+00	.52041E+00	.53792E+00	.35472E-01	.53791E-02
2	.38268E+00	.70711E+00	.67383E+00	.32504E+00	.52041E+00	.53792E+00	.12715E+00	-.55845E+00
1	.92388E+00	.70711E+00	.67383E+00	.88808E+00	.80023E+00	.82701E+00	.26663E+00	-.55207E+00
7	0.	0.	.30320E+00	.32504E+00	.15846E+00	.16339E+00	.35472E-01	.74345E-02
QUESTIONABLE POINT -POOR FIT			.749E-03					
2	.32504E+00	.67383E+00	.57735E+00	.30320E+00	.46210E+00	.47954E+00	.12400E+00	-.55338E+00
2	.88808E+00	.67383E+00	.57735E+00	.67383E+00	.71067E+00	.73490E+00	.26207E+00	-.55116E+00
8	.32504E+00	.30320E+00	.57735E+00	.67383E+00	.46210E+00	.47954E+00	.37339E-01	-.11954E-01

SECTION 3

M N P	X1 Y1 Z1	X2 Y2 Z2	X3 Y3 Z3	X4 Y4 Z4	XP YP ZP	XN YN ZN	A FL CZ1	CZ4 CZ5 CZ6
1	0.	0.	.32504E+00	.38268E+00	.17163E+00	.17887E+00	.12802E+00	-.55519E+00
1	0.	.38268E+00	.32504E+00	0.	.17163E+00	.17887E+00	.27060E+00	-.55928E+00
9	.10000E+01	.92388E+00	.88808E+00	.92388E+00	.93408E+00	.96748E+00	.38023E-01	-.11684E-01
1	.38268E+00	.32504E+00	.67383E+00	.70711E+00	.52041E+00	.53792E+00	.12715E+00	-.55114E+00
2	0.	.32504E+00	.30320E+00	0.	.15846E+00	.16339E+00	.26663E+00	-.55938E+00
10	.92388E+00	.88808E+00	.67383E+00	.70711E+00	.80023E+00	.82701E+00	.35472E-01	.53791E-02
2	0.	0.	.30320E+00	.32504E+00	.15846E+00	.16339E+00	.12715E+00	-.55845E+00
1	.38268E+00	.70711E+00	.67383E+00	.32504E+00	.52041E+00	.53792E+00	.26663E+00	-.55207E+00
11	.92388E+00	.70711E+00	.67383E+00	.88808E+00	.80023E+00	.82701E+00	.35472E-01	.74345E-02
QUESTIONABLE POINT -POOR FIT			.749E-03					
2	.32504E+00	.30320E+00	.57735E+00	.67383E+00	.46210E+00	.47954E+00	.12400E+00	-.55338E+00
2	.32504E+00	.67383E+00	.57735E+00	.30320E+00	.46210E+00	.47954E+00	.26207E+00	-.55116E+00
12	.88808E+00	.67383E+00	.57735E+00	.67383E+00	.71067E+00	.73490E+00	.37339E-01	-.11954E-01

SOLID ANGLE = 12.549

POTENTIAL FLOW PROGRAM SECTION 3

SAMPLE PROBLEM   SPHERE

X VELOCITY=-1.0   Y VELOCITY= 0.0   Z VELOCITY= 0.0

ITERATION	SUM OF CHANGES	A	B1	B2
1	.10255E+00			
2	.19685E-01			
3	.37817E-02			
4	.72593E-03			
5	.13936E-03			

X VELOCITY= 0.0   Y VELOCITY=-1.0   Z VELOCITY= 0.0

ITERATION	SUM OF CHANGES	A	B1	B2
1	.10255E+00			
2	.19685E-01			
3	.37817E-02			
4	.72599E-03			
5	.13936E-03			

X VELOCITY= 0.0   Y VELOCITY= 0.0   Z VELOCITY=-1.0

ITERATION	SUM OF CHANGES	A	B1	B2
1	.10255E+00			
2	.19685E-01			
3	.37817E-02			
4	.72599E-03			
5	.13936E-03			

SAMPLE PROBLEM   SPHERE   PAGE = 1

X FLOW

PT.	X	Y	Z	VX	VY	VZ	ABS.V	CP	SOURCE	V NORMAL
1	.93408	.17163	.17163	-.09689	.26213	.26211	.38315	.85320	.12993	.26E-04
2	.80023	.52041	.15846	-.47232	.66381	.20542	.84020	.29407	.10996	.23E-04
3	.80023	.15846	.52041	-.47232	.20541	.66382	.84020	.29406	.10995	.23E-04
4	.71067	.46210	.46210	-.68728	.52665	.52666	1.01345	-.02708	.09821	.20E-04
5	.17163	.93408	.17163	-1.45656	.26091	.04533	1.48044	-1.19169	.02424	.46E-05
6	.15846	.80023	.52041	-1.47126	.20490	.13186	1.49130	-1.22398	.02159	.47E-05
7	.52041	.80023	.15846	-1.05796	.66240	.13032	1.25500	-.57503	.07160	.15E-04
8	.46210	.71067	.46210	-1.15476	.53123	.34067	1.31595	-.73173	.06417	.13E-04
9	.17163	.17163	.93408	-1.45653	.04530	.26091	1.48041	-1.19162	.02425	.46E-05
10	.52041	.15846	.80023	-1.05799	.13031	.66242	1.25503	-.57509	.07160	.15E-04
11	.15846	.52041	.80023	-1.47125	.13187	.20490	1.49130	-1.22396	.02159	.47E-05
12	.46210	.46210	.71067	-1.15475	.34067	.53122	1.31594	-.73172	.06417	.13E-04

SAMPLE PROBLEM SPHERE PAGE = 2

Y FLOW

PT.	X	Y	Z	VX	VY	VZ	ABS.V	CP	SOURCE	V NORMAL
1	.93408	.17163	.17163	.26091	-1.45653	.04530	1.48041	-1.19160	.02425	.46E-05
2	.80023	.52041	.15846	.66242	-1.05798	.13031	1.25503	-.57509	.07160	.15E-04
3	.80023	.15846	.52041	.20490	-1.47125	.13187	1.49130	-1.22396	.02159	.47E-05
4	.71067	.46210	.46210	.53122	-1.15475	.34067	1.31594	-.73170	.06417	.13E-04
5	.17163	.93408	.17163	.26211	-.09689	.26213	.38315	.85320	.12993	.26E-04
6	.15846	.80023	.52041	.20542	-.47232	.66381	.84020	.29407	.10996	.23E-04
7	.52041	.80023	.15846	.66382	-.47232	.20541	.84020	.29406	.10996	.23E-04
8	.46210	.71067	.46210	.52666	-.68728	.52665	1.01345	-.02708	.09821	.20E-04
9	.17163	.17163	.93408	.04533	-1.45656	.26091	1.48044	-1.19169	.02424	.46E-05
10	.52041	.15846	.80023	.13186	-1.47126	.20490	1.49130	-1.22398	.02159	.47E-05
11	.15846	.52041	.80023	.13032	-1.05796	.66240	1.25500	-.57503	.07160	.15E-04
12	.46210	.46210	.71067	.34067	-1.15476	.53123	1.31595	-.73173	.06417	.13E-04

SAMPLE PROBLEM SPHERE PAGE = 3

Z FLOW

PT.	X	Y	Z	VX	VY	VZ	ABS.V	CP	SOURCE	V NORMAL
1	.93408	.17163	.17163	.26091	.04533	-1.45656	1.48044	-1.19169	.02424	.46E-05
2	.80023	.52041	.15846	.20490	.13186	-1.47126	1.49130	-1.22398	.02159	.47E-05
3	.80023	.15846	.52041	.66240	.13032	-1.05796	1.25500	-.57503	.07160	.15E-04
4	.71067	.46210	.46210	.53123	.34067	-1.15476	1.31595	-.73173	.06417	.13E-04
5	.17163	.93408	.17163	.04530	.26091	-1.45653	1.48041	-1.19160	.02425	.46E-05
6	.15846	.80023	.52041	.13031	.66242	-1.05798	1.25503	-.57509	.07160	.15E-04
7	.52041	.80023	.15846	.13187	.20490	-1.47125	1.49130	-1.22396	.02159	.47E-05
8	.46210	.71067	.46210	.34067	.53122	-1.15475	1.31594	-.73170	.06417	.13E-04
9	.17163	.17163	.93408	.26213	.26211	-.09689	.38315	.85320	.12993	.26E-04
10	.52041	.15846	.80023	.66381	.20542	-.47232	.84020	.29407	.10996	.23E-04
11	.15846	.52041	.80023	.20541	.66382	-.47232	.84020	.29406	.10996	.23E-04
12	.46210	.46210	.71067	.52665	.52666	-.68728	1.01345	-.02708	.09821	.20E-04

SAMPLE PROBLEM SPHERE

ONSET FLOW, VXI= 1.000 VYI= 0.000 VZI= 0.000

LINE NO. 1 PASSING THROUGH QUADRILATERAL 4 WITH STARTING POINT, X= .71000 Y= .45000 Z= .45000

I	X	Y	Z	VX	VY	VZ	CP	K1	K2	H2	SL	V	P
1	.00040	.71015	.70690	1.55673	-.17849	-.19843	-1.49464	.20388	.00941	1.60806	0.00000	1.57944	6
2	.30441	.67883	.66858	1.32555	-.32963	-.33537	-.97822	.38406	.01265	1.47060	.30799	1.40649	6
3	.57188	.57495	.56133	.38308	-.55305	-.53975	-.56364	.78397	.00629	1.23585	.61411	1.25046	8
4	.71889	.45580	.45530	.67737	-.51901	-.51907	.00237	1.20527	-.00313	1.00000	.83107	.99881	4
5	.99755	.00035	-.00042	.00795	-.02041	-.02259	.99901	44.08606	-.31431	.03369	1.55410	.03147	4

SAMPLE PROBLEM SPHERE

ONSET FLOW, VXI= 1.000 VYI= 0.000 VZI= 0.000

LINE NO. 2 PASSING THROUGH QUADRILATERAL 9 WITH STARTING POINT, X= .17000 Y= .17000 Z= .95000

I	X	Y	Z	VX	VY	VZ	CP	K1	K2	H2	SL	V	P
1	-.00000	.17373	.36542	1.56130	-.05335	-.27879	-1.51822	.14174	-.01954	1.02527	0.00000	1.58689	9
2	.16735	.16735	.93566	1.45903	-.04452	-.26151	-1.19915	.15182	-.02186	1.00000	.17010	1.48295	9
3	.35295	.16309	.90213	1.25024	-.07927	-.48451	-.80413	.36648	.00283	.95274	.35875	1.34318	10
4	.69383	.12497	.69405	.78670	-.13556	-.75097	-.20123	.89914	.04870	.74794	.75990	1.09601	3
5	.95979	.00007	.20413	.05112	-.02310	-.30748	.90118	4.43613	-.04801	.23544	1.33970	.31435	3

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